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Control of systems with friction

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Mechanical systems including friction effects constitute a theoretically interesting and practically important class of nonlinear systems. Such systems are commonly described by a second-order nonlinear differential equation with a noncontinuous vector field. Under forcing conditions, a mechanical system with friction can display very complicated behavior, including noninvertible erratic trajectories. In applications, an objective is to induce a prescribed motion in spite of uncertainties (friction effects) and complicated uncontrolled behavior. This objective is attained by means of a control strategy that generates an estimate of the friction forces and counteracts them.

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We consider in this work the model of a mechanical system with friction [1]

$$m\, d^2x/dt^2+\mathcal{F}(x,dx/dt)+\alpha x=\tau_1(t)+\tau_2\ , \qquad (1)$$

where m is the mass of the system, $\tau_1(t)$ is an unknown (regular or stochastic) external force, which may be due to loads and/or noise acting in the mechanism, and τ_2 is a manipulated force used to control the system. The term $\mathcal{F}(x,dx/dt)$ includes all friction effects.

Inaccuracies in the control of mechanical systems are often caused by the presence of friction. Typical errors caused by friction are steady-state errors and tracking lags in position trajectories. Steady-state errors are mainly caused by static (or dry) friction, which is proportional to the velocity direction. Tracking lag errors are generated by viscous friction. To deal with friction, it is necessary to have a good characterization of the structure of the friction model and then to design appropriate compensation techniques [1]. The impact of friction on the performance of precision control systems has received some attention: a number of works have recently appeared discussing modeling and compensation of friction [1–4]. Among the proposed concepts for dealing with friction is to estimate its force and generate a control to counteract it [1–3].

A lot of effort has been devoted to the modeling of friction. It is well established that friction depends on the direction of the movement. Phenomena such as sticking (torque needed to start the motion), as well as downward bends at low velocities have been identified [4]. Detailed experiments [4] performed at low velocities have confirmed Tustin's model which includes a decaying exponential term. In general, it can be said that friction is a complicated phenomenon representing all the forces opposing the motion. Such forces can be both orthogonal and tangential to the direction of motion.

A control strategy to compensate for the presence of friction in the system (1) is proposed in this work. Our contribution in the problem of control is to show that the friction forces can be estimated on line via measurements of position (and velocity) of the mechanical system. The performance of the controller is tested numerically with an oscillator displaying complicated dynamical behavior.

Friction models have been extensively studied [1,2,4]. It is well established that friction forces are functions of the velocity. Although there is disagreement on the character of the functionally of the friction forces with the ve-

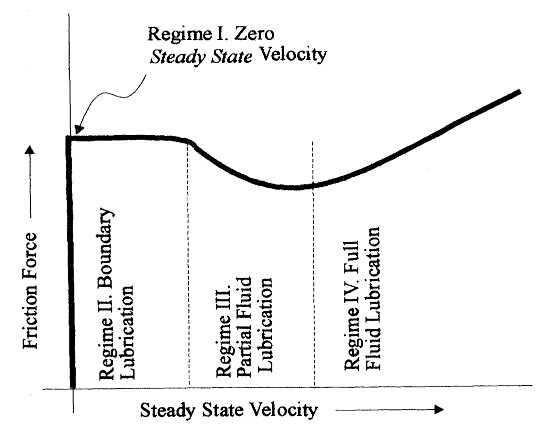

FIG. 1. Friction regimes as a function of the stationary velocity.

locity, experiments have confirmed that, for moderated and low velocities, the main components in the friction forces are caused by the following phenomena [4] (see Fig. 1).

(a) Coulomb friction and stiction. Coulomb friction is due to sticking effects. There is a constant friction torque opposing the motion when the velocity is not zero. For zero velocity, stiction opposes all motions as long as the forces are smaller in magnitude than the stiction force.

(b) Stribeck friction (downward bends). After the stiction force has been surmounted, the friction force decreases exponentially, reaching a minimum, and then increases proportionally with the velocity. These bends occur at velocities close to zero. The friction forces are due to partial fluid lubrication, where the velocity is adequate to entrain some fluid into the junction but not enough to fully separate the surfaces.

(c) Viscous friction. These forces appear at nonzero velocity due to energy dissipation in the lubricant fluid contained between the moving surfaces.

(d) Asymmetries and position dependence. Imperfections and imbalances in the mechanism induce asymmetries and position dependence of the friction forces. However, experiments on industrial mechanisms have shown that this dependence is relatively weak [1,4].

The following expressions are generally used to model the friction effects (a)–(c):

$$F_c(\dot{x})=\beta_1 \mathrm{sgn}(\dot{x}) , \tag{2a}$$

$$F_s(\dot{x})=\beta_2 e^{-\mu|\dot{x}|}\mathrm{sgn}(\dot{x}) , \tag{2b}$$

$$F_v(\dot{x})=\beta_3 \dot{x} , \tag{2c}$$

where β_1 represents the Coulomb friction, and μ represents the slip constant in the Stribeck friction. The friction forces can be written as follows:

$$\mathcal{F}(x,\dot{x})=\phi(x)[F_c(\dot{x})+F_s(\dot{x})+F_v(\dot{x})] . \tag{3}$$

The function $\phi(x)$ is introduced to represent asymmetries and position dependencies of the friction forces or a normal load that may change with displacement. Model (3) can be used for simulating (at least for moderated velocities) real friction effects.

A feature of model (3) is that, because of the function $\mathrm{sgn}(\dot{x})$, it is noncontinuous at $\dot{x}=0$. A consequence of the discontinuity in the term $\mathrm{sgn}(\dot{x})$ is the stick-slip motion [4]. When the trajectory passes through the set $\Sigma=\{\dot{x}=0\}$, the static friction may balance the external forces $\tau_1(t)+\tau_2$. In this situation, the system remains stuck at zero velocity until the driven forces reach the value of the Coulomb friction β_1.

Geometrically, sticking behavior is present when there are some regions in the phase portrait where the flow generated by Eq. (3) is directed from positive velocities ($\dot{x}>0$) toward the set $\Sigma=\{\dot{x}=0\}$, and simultaneously the flow is directed from negative velocities ($\dot{x}<0$) toward Σ, producing a conflict in the flow directions. These regions of conflict are the sticking regions $R\subset\Sigma$. Formally, it is possible to define an equivalent vector field on R which is $(n-2)$-dimensional (zero velocity and consequently, constant position) if the original vector field is n-dimensional. From a control viewpoint, this phenomenon is responsible for the existence of steady-state offsets in position. As a consequence of the reduction of the dimensionality of the flow generated by (3), the dynamical behavior of the system is irreversible (i.e., it is not possible to reconstruct the past of the trajectory by inverting time [in other words, the phase portrait of (3) is not invariant under the transformation $t\to -t$]). The second consequence of the discontinuity of the vector field in (3) is that embedding of an observable variable is not globally diffeomorphic to the phase flow, so that control of the system (3) cannot be attained via delay coordinate techniques [5].

To show that system (3) can display complicated behaviors, let us consider the parameters $\beta_1=1.0$, $\beta_2=\beta_3=0$, the functionality

$$\phi(x)=\begin{cases}1+kx & \text{for } x>-1/k\\ 0 & \text{for } x\le -1/k\end{cases} \tag{4}$$

and the external forces $\tau_1(t)=A\sin(\Omega t)$ and $\tau_2=0$. This case was considered in [6] to study the dynamical effects of oscillations in mechanical systems with dry friction. The function $\phi(x)$ represents a normal load which varies with the position. Note that $\phi(x)$ is continuous, such that the only source of discontinuities is the term $\mathrm{sgn}(\dot{x})$. Figure 2 shows the $(x,\dot{x})$ portrait for the case $\Omega=1.25$, $A=1.9$, and $k=1.5$. One can think of Fig. 1 as being the phase portrait of the system $d^2x/dt^2+\mathcal{F}(x,dx/dt)+\alpha x=0$ perturbed by an external signal $\tau_2(t)$. In the unforced phase portrait, two regions R and S in the set Σ (with $R\cup S=\Sigma$) are distinguished. In the set $S=\{\dot{x}=0,x\le\gamma^*\}$ for certain $\gamma^*<0$, the direction of both vectors agrees, such that the function $\mathrm{sgn}(\dot{x})$ switches only one time. On the other hand, the sticking phenomenon appears in the set $R=\{\dot{x}=0,x>\gamma^*\}$ where the flow of the system is in conflict. The segment of line $R\setminus\{0\}$ is a set of *weak* equilibrium points because a small perturbation in the vector field *slides* the trajectories toward the *strong* equilibrium point (0,0). When the flow of the system is perturbed with a harmonic force $\tau_1(t)=A\sin(\Omega t)$, almost all the points of the set S remain as switching points. This invariance is due to the transversality of the trajectories of (3) in S. On the other

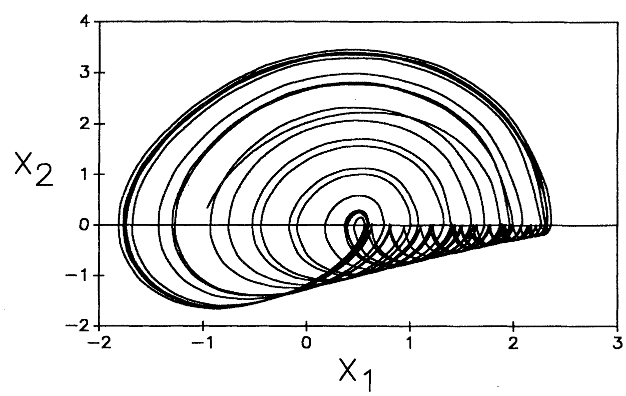

FIG. 2. Phase portrait of the system (3) under the functionality (4) and an external force $\tau_1(\tau)=A\sin(\Omega t)$.

hand, the set R induces a band $\mathcal{B}$ of *sliding* flow (small velocity dynamics with alternating stiction periods), where the trajectories move to regions with a lower normal load $\phi(x)$. The trajectory leaves the *sliding* band $\mathcal{B}$ in a region where the force $\tau_1(t)$ becomes larger than the Coulomb friction and returns again to the sliding band $\mathcal{B}$.

Complicated behaviors, such as the one discussed above, are undesirable in most cases because degradation of mechanical parts may appear. Thus, control actions are necessary in order to induce more regular behaviors despite friction and external forces.

The objective of control is to regulate the behavior of system (1). Such an objective is attained by applying a force to compensate for the external $\tau_1(t)$ and friction forces $\mathcal{F}(x,\dot{x})$. In this way, the system can be controlled to generate a prescribed behavior.

Assume the following.

(i) Position x (and, if possible, velocity $\dot{x}$) is available for measurement.

(ii) The parameter α and the mass m are exactly known. The friction forces $\mathcal{F}(x,\dot{x})$ are not known.

(iii) $\tau_1(t)$ is bounded for all $t \geq 0$.

Some comments regarding the assumptions are in order. Assumption (i) is physically realizable through shaft encoder measurements. Except for robots, the mass m is constant in most mechanical systems, so that it can be accurately known. On the other hand, α can be interpreted as a string constant, which is well known in most cases. That friction effects $\mathcal{F}(x,\dot{x})$ are not well known is a realistic situation for practical applications.

Without lost of generality, assume that $m=1$. Write system (1) as a set of first-order equations:

$$\begin{aligned} \dot{x}_1 &= x_2 , \\ \dot{x}_2 &= -\alpha x_1 - \mathcal{F}(x_1,x_2) + \tau_1(t) + \tau_2 . \end{aligned} \tag{5}$$

Let $y(t) \in \mathbb{C}^2$ be a prescribed position trajectory. If the friction forces are known, the following control feedback:

$$\begin{aligned} \tau_2^I(x,t) &= -\tau_1(t) + \alpha x_1 + \mathcal{F}(x_1,x_2) + v , \\ v &= d^2y/dt^2 + g_1(x_2 - dy/dt) + g_2(x_1 - y) , \end{aligned} \tag{6}$$

steers asymptotically the system trajectories to a behavior with $x_1(t) = y(t)$. The control parameters g_1, g_2 are chosen in such a way that the matrix

$$M = \begin{bmatrix} 0 & 1 \\ -g_2 & -g_1 \end{bmatrix}$$

be stable. The control feedback (6) will be called "ideal control feedback" because it assumes perfect knowledge of friction forces. The main advantage of using (6) is that the controlled dynamics are linear. When the friction forces $\mathcal{F}(x_1,x_2)$ are not known, an alternative is to use an approximation of the control (6) to obtain a controlled behavior as close as possible to a linear one.

Let $\psi(t) = -\mathcal{F}[x_1(t),x_2(t)] + \tau_1(t)$. Then, system (5) can be written as

$$\dot{x}_1 = x_2 , \tag{7a}$$

$$\dot{x}_2 = -\alpha x_1 + \psi(t) + \tau_2 . \tag{7b}$$

In principle, the term $\psi(t)$ can be calculated via the acceleration $a(t) = d^2x/dt^2$ Eq. (7b):

$$\psi(t) = a(t) + \alpha x_1 - \tau_2 . \tag{8}$$

That is, the friction plus exogenous forces can be estimated via a balance of forces. To synthesize a controller with an estimation like (8), we have to use approximations for the acceleration $a(t)$. Let $\Delta > 0$ and consider the control law in the time interval $t \in [t_k, t_{k+1})$, $t_{k+1} - t_k = \Delta$:

$$\begin{aligned} \tau_2(x,t) &= -\bar{\psi}_k + \alpha x_1 + v , \\ v &= d^2y/dt^2 + g_1(x_2 - dy/dt) + g_2(x_1 - y) , \end{aligned} \tag{9}$$

$$\bar{\psi}_k = a_k + \alpha x_{1,k} - \tau_{2,k} , \tag{10}$$

where x_2 is the measured or estimated velocity, $a_k = a(t_k)$ is the acceleration of the system at $t = t_k$, and $\tau_{2,k} = \tau_2(t_k)$ is the force applied to control the mechanical system. Note that as $\Delta \to 0$, the estimate (10) converges to $\psi(t)$ in (8). If $\bar{\psi}_k \to \psi$, then $\tau_{k,2} \to \tau_2$ and from (9) and (10)

$$\tau_2[x(t)] = a(t) + \tau_2(t) + v ,$$

from where $a(t) = v$. Consequently, in the limit as $\Delta \to 0$ the behavior under control (9),(10) converges to the behavior under the ideal control feedback (6). Control (9) can be interpreted as a Δ perturbation of the ideal control feedback (6). Evidently, the case $\Delta = 0$ cannot be implemented because in the limit $\Delta \to 0$ control (9),(10) is not well defined.

The estimation procedure (10) has a nice interpretation. The combination of (9) and (10) yields

$$\bar{\psi}_k = a_k - v_k + \bar{\psi}_{k-1} .$$

On the other hand, (7) and (9) lead to the following dynamics:

$$a_k = v_k + (\psi_k - \bar{\psi}_k), \quad t \in [t_{k-1}, t_k) ,$$

so therefore

$$\bar{\psi}_k = (\psi_k + \bar{\psi}_{k-1})/2 = 0.5\bar{\psi}_{k-1} - 0.5\psi_k . \tag{11}$$

From (11), one can see that the estimate $\bar{\psi}_k$ is the arithmetic average of the actual friction forces ψ_k and the last estimate $\bar{\psi}_{k-1}$. As $\Delta \to 0$, $\bar{\psi}_{k-1} \to \psi_k$ and $\bar{\psi} \to \psi$. Thus, in the limit $\Delta \to 0$ the estimate $\bar{\psi}$ converges to the actual term ψ. On the other hand, Eq. (11) can be interpreted as a contraction mapping, with a decaying rate equal to 0.5. If the friction forces ψ are bounded, the estimator (10) yields bounded estimates $\bar{\psi}_k$. Therefore, if the friction forces are bounded, the estimation procedure (10) is stable. Under the control (9) the system becomes

$$\dot{z} = Mz + \eta(t) ,$$

where $z_1 = x_1 - y$, $z_2 = x_2 - dy/dt$, and $\eta(t) = [0, \psi - \bar{\psi}]$. It is not hard to show that the asymptotic error $||z(t \to \infty)||$ is proportional to the error estimation $|\psi - \bar{\psi}|$. Consequently, the better the estimation $\bar{\psi}$, the lower the control error $||z||$.

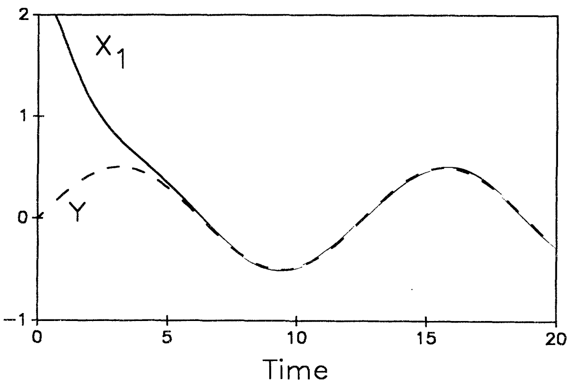

FIG. 3. Dynamics of the controlled system (3) under the feedback control (9),(10).

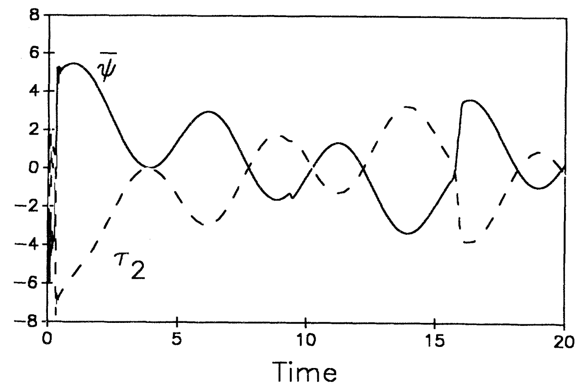

FIG. 4. Estimated friction forces $\overline{\Psi}_k$ and applied control force τ_2 derived from the controlled system (3),(9),(10).

To test the performance of control (9),(10), system (3) with normal load (4) and $\tau_1(t)=A\sin(\Omega t)$ (the oscillator displaying the behavior in Fig. 1) will be used. Assume that the position x_1 and the velocity x_2 are available for measurement at sampling rates Δ. To implement estimation (10), the acceleration a_k is approximated as a finite difference: $a_k \cong (x_{2,k}-x_{2,k-1})/\Delta$, such that an error $O(\Delta)$ in the approximation of the acceleration is introduced. We take $\Delta=0.01$, which implies that the measurements are made at frequencies of 100 Hz. Assume that the desired position trajectory is $y(t)=B\sin(\overline{\Omega}t)$. Figure 3 shows the convergence of the controlled system to the position trajectory $y(t)$. The values $g_1=2$ and $g_2=1$ have been set such that matrix M has eigenvalues $\{-1,-1\}$. Figure 4 shows the estimated friction forces $\overline{\psi}_k$ and the applied control force τ_2. Note that the applied force τ_2 counteracts the friction forces ψ (i.e., $\tau_2 \cong \psi$) after a transient behavior.

Suppose now that only the position x_1 is available for measurements. In this case we use stable finite differences to estimate acceleration: $a_k \cong (x_{1,k}-2x_{1,k-1}+x_{1,k-2})/\Delta^2$. In addition, the velocity is estimated via $x_{2,k} \cong (x_{1,k}-x_{1,k-1})/\Delta$. Also for $\Delta=0.01$, the behavior under these conditions is analogous to the one shown in Fig. 3.

In summary, a strategy for controlling systems with friction was studied. In most precision mechanical systems, such as telescopes, friction is an important effect that must be considered in order to obtain smooth sliding between mechanical parts. To compensate for friction forces, the control strategy makes use of friction force estimates derived from a balance of forces. The performance of the controller was tested numerically in a system with variable normal load showing good dynamical performance.

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